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Date: 1998 February 9 TM98-02
To: Distribution
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Subject: GAMES sensitivity II. Effect of the basic angle

I. INTRODUCTION

This memorandum is the second in a series investigating the dependence of GAMES performance on a variety of design parameters affecting the observing geometry. These studies all consider the first stage of data analysis (modeling the spacecraft rotation from "batches" of astrometric data) and rate the instrument's performance by how well it ties together the swath of sky seen during a "batch interval" (of order one day). In the first memorandum (Chandler and Reasenberg 1998, hereafter TM98-01), we described the effect of varying the complexity of the rotation model and concluded that avoiding frequent attitude corrections ("rotation breaks") is an important consideration in increasing the scientific output of the mission. The present study examines the effect of varying the angle between the instrument's two fields of view (the "basic" or "opening" angle). The remainder of this memorandum describes the configurations used in this study (Section II), presents the results from a series of simulations (Section III), and draws conclusions about spacecraft design considerations (Section IV). A glossary of terms is included as Appendix A.

II. OPERATIONAL MODEL

Refer to TM98-01, Sections II - IV, for a description of the spacecraft model and of the methods used for simulating observations and reducing the simulated data. The present study differs from that of TM98-01 in that only a single value of the spacecraft precession rate is used, namely, 6 deg/d, which yields a minimum of 50% overlap of the observing spiral band between successive instrument rotations. In addition, most of the tests use the same degree of complexity in the rotation model: for each span, there are 5 ϕ , 2 α, and 2 δ coefficients. However, we have also included one set of runs with 95 ϕ coefficients in a single span (i.e., with no rotation breaks). As in TM98-01, runs with rotation breaks are repeated with 16 different randomly chosen sets of span lengths, and the results are averaged to suppress the statistical noise from the span lengths. In runs with a single rotation span, there is no random element in the input and no need to average multiple runs.

We have introduced a new variable into the process of choosing random rotation span lengths, namely, the standard deviation of the Gaussian distribution. (Previously, this quantity was fixed at one quarter of the mean span length.) In most of the runs, it remains as before, but we include some runs with a narrower spread. We also include runs with a new average span length, one rotation period (2 hr).

The main concern of this study is the dependence of mission output upon the basic (i.e., opening) angle. Consequently, the simulations come in sets with values of the basic angle in steps of 10 deg, and most of these sets span the range from 0 to 180 deg (even though the 0 deg case is degenerate). In practice, the simulation software uses algorithms that require the basic angle to differ from both extremes (0 and 180 deg) by at least the angular offset between the two rows of detector elements (taken to be 0.1 deg in this study). Therefore, the endpoints of the studied range have been set *arbitrarily* to 0.15 deg and 179.5 deg.

III. RESULTS

The same figure of merit is used in this study as in TM98-01. As before, the cohesion of the rotation model is gauged by an average of the uncertainty in $\Delta\phi$, the modeled difference in ϕ between pairs of epochs. See Section V of TM98-01 for the details.

The main results of this study are presented in Figure 1. Each part of the figure shows a summary of a full set of runs at different basic angles. Though they differ in detail, they present a remarkably consistent picture over a variety of conditions. In each, there is a broad minimum in the uncertainty, punctuated by a few "bad" angles, and the angles near 0 and 180 deg are particularly bad. The differences among the figures can be characterized largely by (1) the depth of the minimum, (2) the relative badness of the few bad angles, and (3) the degree of asymmetry about 90 degrees. Not all the bad angles appear in each figure. For example, 60 deg shows no discernible peak, except in Figure 1b. Indeed, Figure 1c has only one bad angle (90 deg), and that is just barely perceptible. Figure 1d is the only case where any of the bumps in the middle rivals the extremes near 0 and 180 deg. Interestingly, Figure 1d has a small bump at 70 deg; a separate run (not plotted in the figure) with a 72 deg basic angle shows a higher (but only slightly higher) uncertainty than 70 deg. Other runs for exactly 1/7 and 1/8 of 360 deg show no hint of bumps at those angles.

It is easy to understand the existence of intrinsically bad basic angles. The mechanism is slightly different between the case of frequent rotation breaks and that of a single rotation span, but the potentially bad angles are the same in both cases. By observing the same star through each field of view, the instrument ties together the rotation model with many links offset by the basic angle. When the angle is commensurate with a complete rotation (as in 60, 72, 90, 120, and 180 deg), these many links repeatedly tie the same "things" together into nearly disjoint subsets, but without linking the batch as a whole, i.e., without including links between the subsets. (Because of the overlap of the observing spiral band on successive rotations, there are already many such links at 360 deg, especially within the two regions where the overlap is nearly complete. In these regions, there are also links at all multiples of 360 deg up to the full length of a batch interval.)

In the case of frequent rotation breaks, the "things" to be linked together are the independent models covering the different spans; for the case of a single span with many model coefficients to be estimated, the "things" are the signatures of the coefficients. Since the rotation breaks are not spaced uniformly, the "poisonous" effect

Table 1. Mean log uncertainty in $\Delta\phi$ (arcsec) as a function of basic angle, with two Monte Carlo skies

Basic angle (deg)	First sky (log)	Alternate sky (log)	Ratio
50	−2.925	−2.925	1.000
60	−2.956	−2.958	1.005
70	−2.975	−2.983	1.019
80	−2.990	−3.000	1.023
90	−2.986	−2.997	1.026
100	−2.998	−3.009	1.026
110	−2.992	−3.004	1.028
120	−2.951	−2.961	1.023
130	−2.963	−2.969	1.014

of commensurability is smeared out by somewhat more than the average width of a rotation span (60 deg in Figures 1a and 1b, 360 deg in Figure 1c). This explains why the bump at 60 deg is scarcely noticeable and why it is more noticeable in Figure 1b (with more nearly uniform span lengths) than in Figure 1a. It also explains why 120 and especially 90 are much less severe than 180 and why the bump at 180 is so much reduced in Figure 1c. A corresponding mechanism operates in Figure 1d to suppress the badness at the special angles smaller than 90 deg – the effective angular resolution (in inverse cycles) of the rotation model is given approximately by the number of ϕ coefficients divided by four times the number of rotations. (We have verified that reducing the number of coefficients from 95 to 74 suppresses the bump at 90 deg, and further reducing the number from 74 to 59 greatly reduces the bump at 120 deg as well. The results are otherwise very similar to Figure 1d and are not shown here.)

Another concern we address in this study is the sensitivity of the results to the particular set of randomly positioned stars. Although an exhaustive test could examine many different sets of stars, we see no need to go to such lengths; the sets are so large (about 2000 stars each) that a great deal of smoothing occurs naturally. In fact, we examine only one additional set, and the results of the test show that no more are needed. For this comparison, we have repeated the runs across the broad minimum in Figure 1a with a separate set of Monte Carlo stars (with a uniform probability density, as before). The results for the two sets of stars are shown in Table 1. The two profiles track each other closely across this whole range, differing by less than 3% (all in the same direction and smoothly varying with basic angle). This, then, is the scale of statistical fluctuations to be expected within the context of uniform star densities. We conclude that the behavior we see is intrinsic and insensitive to the exact positions of the observed stars.

Two illustrative examples are shown in Figure 2, revealing the structure of the model cohesion as a function of lag angle. Figure 2a shows the periodic performance characteristic of a spinning astrometric instrument. Lags close to a whole number of rotations have a noticeably better cohesion than any others. Figure 2b shows much the same picture, but with a small additional periodicity of the basic angle (120 deg), revealed because of the commensurability with a complete rotation.

IV. DISCUSSION

The similarities of the plots in Figure 1 lead to several conclusions about the design of the instrument. Most obviously, the "bad" basic angles should be avoided if possible. On the other hand, the bad angles are so few that avoiding them can hardly be a difficult goal. More to the point, there is very little difference in performance over a wide range of basic angle, so that other considerations can govern the choice.

We have not looked with high resolution *around* one of the bad angles to discover what sort of "fine structure" appears. There is presumably some structure associated with the dual-strip detector, but it may be smeared out by the same mechanisms that suppress the spikes at angles below a configuration-dependent cutoff. If reasons are discovered that strongly favor a basic angle close to one of the bad angles, it will be necessary to explore that region in fine detail, but the only constraints known at present are broad ones.

As in TM98-01, it is clear that frequent rotation breaks should be avoided if at all possible. Even the worst basic angle in Figure 1d (0 deg, no rotation breaks) shows a better cohesion than the best angle in Figures 1a and 1b (100 deg, standard rotation breaks). More realistically, a comparison of like to like shows an improvement by about a factor of ten from 1a to 1d, even though the total number of parameters is roughly comparable. Not surprisingly, the intermediate case represented by Figure 1c, with an average rotation span of 2 hr, instead of 20 min as in 1a and 1b or 12 hr as in 1d, displays a cohesion that is also intermediate between those two extremes. Thus, any reduction in the frequency of correction events can be expected to improve the instrument performance.

V. REFERENCES

Chandler, J. F. and Reasenberg, R. D. 1998, "GAMES sensitivity to complexity of rotation," TM98-01

Appendix A. GLOSSARY

Basic angle: the angle between the centers of the instrument's two fields of view. See also "opening angle."

Batch interval: the period during which data are collected for a single (first stage) analysis.

Cross-scan direction: the direction on the sky perpendicular to the time-averaged direction of motion of the center of a field of view of the instrument; alternatively, the direction in the detector plane perpendicular to the time-averaged motion of a star image in the center of the field of view. (The center of the detector area may actually be obscured, but it remains the logical reference point.) More generally, the concept may be extended to refer to the un-averaged, instantaneous motion or the motion of a point away from the center. The direction local to a particular point in the field may differ from that of the center because of optical distortion.

Observing spiral: the path on the celestial sphere of the center of one of the instrument's fields of view during a batch interval. The two observing spirals are conceived to be very nearly coincident, except at the non-overlapping ends.

Observing spiral band: the region of the celestial sphere covered by one instrument field of view during a batch interval. Since the two spirals are nearly coincident, this term may also refer to the region of sky covered by both fields of view.

Opening angle: the angle between the centers of the instrument's two fields of view. See also "basic angle," the term used by the HIPPARCOS team.

Prereduction: a technique of speeding up least-squares parameter estimation by eliminating the "uninteresting" parameters from the normal equations. To within the numerical accuracy of computing, this technique gives exactly the same results (estimates and covariances) for the interesting parameters as would be obtained from a complete solution.

Rotation break: an attitude control event.

Rotation span: a period during which the spacecraft rotates without attitude control events. (The rotation spans are separated by attitude control events.)

Scan direction: the direction on the sky of the time-averaged motion of the center of a field of view of the instrument; alternatively, the direction in the detector plane of the time-averaged motion of a star image at the center of the field of view. See the discussion of variants under "Cross-scan direction."

FIGURE CAPTIONS

Figure 1. (a) Geometric mean uncertainty in the difference in ϕ between pairs of points on the evenly-spaced grid, averaged over all lags from 0.11 to 0.5 of the batch interval. Each point represents an average of 16 runs, each with an independent set of Gaussian-distributed rotation spans of 20 min average length and 5 min standard deviation. Each span has a separate rotation model consisting of 5 coefficients for ϕ, 2 for α, and 2 for δ.

(b) Same as (a), except that the distribution of rotation spans has a standard deviation of 1 min.

(c) Same as (a), except that the distribution of rotation spans has a mean of 2 hr and a standard deviation of 30 min.

(d) Similar to (a), but each point represents a single run with no rotation breaks. The overall rotation model has 95 coefficients for ϕ, 2 for α, and 2 for δ.

Figure 2. (a) Geometric mean uncertainty in the difference in ϕ between pairs of points on the evenly-spaced grid, as a function of lag for a basic angle of 100 deg (corresponding to the minimum of Figure 1a). The lags are given in hundredths of the batch interval.

(b) Similar to (a), but with a basic angle of 120 deg (corresponding to a bump in Figure 1b).

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